

FEB 2023 EXAMINATION
B.E. /B.TECH EXAM
MA 10001:-MATHEMATICS-I

Time: 3 Hrs.]

[Max. Marks: 70

TOTAL NO. OF QUESTIONS IN THIS PAPER:5

Note: Attempt all questions. All questions carry equal marks. Each question carries five subparts a, b, c, d and e. Parts a, b and c are compulsory and attempt any one from d and e.

S. No.	Questions	Marks	CO	BL	PI
Q.1	(a) Write down the statement of Taylor's theorem for one variables.	(02)	CO1	1	1.1.1
	(b) If $u = x(1-y), v = xy$ then find the value of $\frac{\partial(u,v)}{\partial(x,y)}$.	(02)	CO1	2	1.1.1
	(c) If $z = f(x,y), x = e^u + e^{-v}, y = e^{-u} - e^v$ then show that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y}$	(03)	CO1	2	1.1.1
	(d) Expand $e^{ax} \sin by$ in powers of x & y as far as the term of third degree using Maclaurin's theorem.	(07)	CO1	3	1.1.1
	OR				
	(e) If $u = \sin^{-1} \left[\frac{x+y}{\sqrt{x} + \sqrt{y}} \right]$ then prove that (i) $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \tan u$ (ii) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{-\sin u \cos 2u}{4 \cos^3 u}$	(07)	CO1	3	1.1.1
Q.2	(a) Define curvature of a curve.	(02)	CO2	1	1.1.1
	(b) Write down the steps to find maximum value for any function of two variables.	(02)	CO2	1	1.1.1
	(c) Find the radius of curvature for the curve $y = \frac{a}{2} (e^{x/a} + e^{-x/a})$ at the point (x, y) .	(03)	CO2	2	1.1.1
	(d) A Rectangular box, open at the top, is to have a volume of 32 cc. Find the dimensions of the box requiring least material for its construction.	(07)	CO2	4	1.1.1
	OR				
	(e) Find the asymptotes of the curve $x^3 + 2x^2y - xy^2 - 2y^3 + xy - y^2 - 1 = 0$	(07)	CO2	2	1.1.1
Q.3	(a) Prove that $\int_0^1 \frac{x^2}{\sqrt{1-x^5}} dx = \frac{1}{5} \beta \left(\frac{3}{5}, \frac{1}{2} \right)$	(02)	CO3	2	1.1.1
	(b) Evaluate $\int_0^2 \int_0^2 (x^2 + y^2) dx dy$	(02)	CO3	2	1.1.1

	(c)	Evaluate $\int_0^{\infty} \sqrt{x} e^{-3\sqrt{x}} dx$	(03)	CO3	2	1.1.1
	(d)	Evaluate the following integral by changing the order of integration $\int_{x=0}^{2a} \int_{y=\frac{x^2}{4a}}^{3a-x} dx dy$	(07)	CO3	3	1.1.1
		OR				
	(e)	Evaluate $\iint_R \frac{r dr d\theta}{\sqrt{a^2 + r^2}}$ over one loop of the curve $r^2 = a^2 \cos 2\theta$	(07)	CO3	3	1.1.1
Q.4	(a)	Find the maximum value of r for the curve $r = a + b \cos \theta, a < b$	(02)	CO4	2	1.1.1
	(b)	Find point of intersection with coordinate-axis for the curve $y = x(x^2 - 1)$.	(02)	CO4	2	1.1.1
	(c)	Find the area bounded by the parabola $y = x^2$ and the line $y = x$	(03)	CO4	3	1.1.1
	(d)	Find the volume of the solid formed by the revolution of the cardioid $r = a(1 + \cos \theta)$ about initial line.	(07)	CO4	4	1.1.1
		OR				
	(e)	Find the whole length of the curve $8a^2 y^2 = x^2(a^2 - x^2)$	(07)	CO4	3	1.1.1
Q.5	(a)	In Boolean Algebra $[B, +, \cdot, ']$ prove that $a + 1 = 1$.	(02)	CO5	2	1.1.1
	(b)	In Boolean Algebra $[B, +, \cdot, ']$ prove that $a \cdot b + a' \cdot b' = (a' + b)(a + b')$.	(02)	CO5	2	1.1.1
	(c)	In Boolean Algebra $[B, +, \cdot, ']$ simplify the following $[a + (a' \cdot b)][a' + (a \cdot b)]$	(03)	CO5	2	1.1.1
	(d)	(i) Replace the given function by a simple switching circuit. $F(x, y, z) = (x + y)(x + z) + z \cdot (x + y \cdot z)$ (ii) State and prove Involution law in Boolean algebra.	(04+03)	CO5	3	1.1.1
		OR				
	(e)	Change the Boolean function $F(x, y, z) = (x + y + z) \cdot (xy + x'z)$ into conjunctive and disjunctive normal form.	(07)	CO5	2	1.1.1

JUNE-JULY 2022 EXAMINATION

B.E. /B.TECH EXAM

MA 10001:-MATHEMATICS-I

Time: 3 Hrs.]

[Max. Marks: 70

TOTAL NO. OF QUESTIONS IN THIS PAPER:5

e: Attempt all questions. All questions carry equal marks. Each question carries five subparts a, b, c, d and e. Parts a, b and c are compulsory and attempt any one from d and e.

S. No.	Questions	Marks	CO	BL	PI
Q.1	(a) Write down the statement of Maclurin's theorem for function of two variables.	(02)	CO1	1	1.1.1
	(b) If $u = e^x \sin y, v = x + \log \sin y$ then find the value of $\frac{\partial(u,v)}{\partial(x,y)}$.	(02)	CO1	1,2	1.1.1
	(c) If $u = (1 - 2xy + y^2)^{-1/2}$ then show that $x \frac{\partial u}{\partial x} - y \frac{\partial u}{\partial y} = u^3 y^2$.	(03)	CO1	2	1.1.1
	(d) Find the first four terms in the expansion of $\log_e \sin(x+h)$ in the ascending powers of h by Taylor's theorem. Hence find the value of $\log_e \sin 31^\circ$ to four places of decimal.	(07)	CO1	3	1.1.1
	OR				
	(e) If $u = \sin^{-1} \left[\frac{x+y}{\sqrt{x} + \sqrt{y}} \right]$ then prove that i) $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \tan u$ ii) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -\frac{\sin u \cos 2u}{4 \cos^3 u}$	(07)	CO1	2	1.1.1
Q.2	(a) Find the radius of curvature of the curve $s = c \tan \psi$. at the point (s, ψ) .	(02)	CO2	2	1.1.1
	(b) Find the stationary point of the function $f(x, y) = 4x^2 + 9y^2 - 8x - 12y + 4$	(02)	CO2	1,2	1.1.1
	(c) Find the co-ordinate of center of curvature of the parabola $y^2 = 4ax$ at the point $(a, 2a)$.	(03)	CO2	2	1.1.1
	(d) A rectangular box open at the top is to have a volume of 32 cc. find the dimension of the box requiring least material for its construction.	(07)	CO2	3	1.1.1
	OR				
	(e) Find the asymptotes of the curve $4x^3 - x^2y - 4xy^2 + y^3 + 3x^2 + 2xy - y^2 - 7 = 0$	(07)	CO2	2	1.1.1
Q.3	(a) Evaluate $\int_0^{\infty} x^n e^{-\sqrt{ax}} dx$	(02)	CO3	2	1.1.1

	(b)	Prove that $\int_0^1 \frac{x^2}{\sqrt{1-x^5}} dx = \frac{1}{5} \beta\left(\frac{3}{5}, \frac{1}{2}\right)$	(02)	CO3	2	1.1.1
	(c)	Evaluate $\int_0^4 \int_0^x x^2 dy dx$.	(03)	CO3	2	1.1.1
	(d)	Evaluate the following integral by changing the order of integration $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x dy dx}{\sqrt{x^2+y^2}}$	(07)	CO3	2	1.1.1
	OR					
	(e)	Evaluate $\iint_R r^3 dr d\theta$ over the area included between the circles $r = 2a \cos \theta$ & $r = 2b \cos \theta, b < a$.	(07)	CO3	2	1.1.1
Q.4	(a)	write down the formula for surface area of the solid generated by revolution about the x-axis, of the area bounded by the curve, the x-axis and the ordinates $x = a, x = b$.	(02)	CO4	1	1.1.1
	(b)	Find the symmetricity of the curve $y^2(a^2+x^2) = x^2(a^2-x^2)$ and also find point of intersection with x-axis.	(02)	CO4	1	1.1.1
	(c)	Find the area included between the parabola $y^2 = 4ax$ and its latus rectum.	(03)	CO4	2	1.1.1
	(d)	Find the volume of the solid formed by the revolution of the curve $xy^2 = 4(2-x)$ through four right angles about the y-axis.	(07)	CO4	3	1.1.1
	OR					
	(e)	Find the whole perimeter of the cardioids $r = a(1 + \cos \theta)$	(07)	CO4	2	1.1.1
Q.5	(a)	Define minimal Boolean function with example.	(02)	CO5	1	1.1.1
	(b)	In Boolean Algebra $[B, +, \cdot, ']$ prove that $a + a = a$.	(02)	CO5	1	1.1.1
	(c)	In Boolean Algebra $[B, +, \cdot, ']$ prove that $(x + y \cdot z)(y' + x)(y' + z') = x \cdot (y' + z')$	(03)	CO5	2	1.1.1
	(d)	Replace the switching function $F(a, b, c) = a \cdot c + b \cdot (b' + c) \cdot (a' + b \cdot c')$ by a simple switching circuit and draw the circuit for the same.	(07)	CO5	3	1.1.1
	OR					
	(e)	Change the Boolean function $F(x, y, z) = (x + y) \cdot (x + z') + (y + z')$ into disjunctive normal form.	(07)	CO5	2	1.1.1

2/2

Time : 90 Minutes]

[Max. Marks :40

TOTAL NO. OF QUESTIONS IN THIS PAPER : 5

Note: Each question carry three subparts a, b and c. Attempt any two from a , b and c. All questions carry equal marks.

Questions		Marks	COS	BL	PI
Q.1 (a)	(i) If $u = \tan^{-1}\left(\frac{y^2}{x}\right)$ show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \sin 2u$ (ii) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -\sin^2 u \sin 2u$	04	CO1	1,2	1.1.
(b)	If $u = x + y^2$, $v = y + z^2$, $w = z + x^2$ prove that $\frac{\partial x}{\partial u} = \frac{1}{1+8xyz}$	04	CO1	1,2	1.1.
(c)	Expand $e^x \cos y$ in power of x & y as far as the terms of the third degree.	04	CO1	1,2	
Q.2 (a)	A rectangular box is placed on xy plane whose vertex is at the origin. Find the maximum volume of the box if the vertex facing to the origin lies on the plane $6x + 4y + 3z = 24$	04	CO2	1,2	1.1.
(b)	Find the equation of the circle of curvature of the rectangular hyperbola $xy = 12$ at the point $(3, 4)$	04	CO2	1,2	1.1.
(c)	Find the asymptote of the curve $(2x - 3y + 1)^2(x + y) - 8x + 2y - 9 = 0$	04	CO2	1,2	1.1.
Q.3 (a)	Show that $\int_0^2 x(8 - x^3)^{\frac{1}{3}} dx = \frac{16\pi}{9\sqrt{3}}$	04	CO3	2,3	1.1.
(b)	Transform the integral $\int_0^a \int_0^{\sqrt{a^2-x^2}} y^2 \sqrt{x^2 + y^2} dx dy$ by changing to polar coordinate and hence evaluate it.	04	CO3	2,3	1.1.
(c)	Change the order of integration in $\int_0^1 \int_{e^x}^e \frac{dx dy}{\log y}$ and hence evaluate the same	04	CO3	1,2	1.1.
Q.4 (a)	The cardioid $r = a(1 + \cos\theta)$ revolves about the initial line. Find the volume of the solid generated.	04	CO4	1,2	1.1.

(b)	Find the length of the curve of the semicubical parabola $ay^2 = x^3$ to the point (a, a)	04	CO4	1,2	1.1.
(c)	Trace the curve $y^2(x^2 + y^2) + a^2(x^2 - y^2) = 0$	04	CO4	1,2	1.1.
Q.5	Change the following Boolean function into disjunctive normal form	04	CO5	1,2	1.1.
(a)	$f(x, y, z) = [x + (x' + y)'] \cdot [x + (y' \cdot z)']$ is				
(b)	Draw a circuit for the following Boolean function and replace it by a simpler one $f(x, y, z) = x \cdot z + [y \cdot (y' + z) \cdot (x' + x \cdot z)']$	04	CO5	1,2	1.1.
(c)	Prove the following: (i) $[a + (a' + b)'] \cdot [a + (b' \cdot c)'] = a$ (ii) $(a + b') \cdot (a' + b) \cdot (a' + b') = a' \cdot b'$	04	CO5	1,2	1.1.
